Chapter 6

INDUCED CURRENTS, EQUIVALENT NETWORKS, AND GAIN-BANDWIDTH PRODUCT

In the application of electron tubes as circuit elements we are interested in the currents that flow in the circuits external to the tube as a result of electron motion in the space between the electrodes. In this chapter we show how the external currents are related to the electron currents in the interelectrode space of a tube for cases in which the period of the ac voltages applied to the electrodes is long compared with the time taken by the electrons to travel between the electrodes.

When an electron is gaining kinetic energy under the influence of an electric field, the source that provides the field loses an equal amount of energy. Similarly, when an electron is slowed down by an electric field, an amount of energy equal to the kinetic energy lost by the electron appears elsewhere in the system. Several problems that illustrate these effects are discussed in Sections 6.1 and 6.2.

In Section 6.3 we consider the small-signal analysis of simple circuits using grid-controlled tubes. We shall find that a grid-controlled tube operated with negative bias on the control grid and driven by a small ac signal can be simulated by either of two networks. One network contains a constant current generator and passive elements, such as resistances, capacitances, and inductances, whereas the second contains a constant voltage generator and passive elements. In analyzing the small-signal behavior of an amplifier stage, these networks can be substituted in place of the tube, and the currents that flow in the various circuit elements can be determined by simple application of Kirchhoff's Laws.

By applying the networks to the analysis of an amplifier stage which is part of a multistage amplifier, we find that the product of gain and bandwidth that can be obtained from the stage is a constant that depends only on parameters of the tube itself and on the capacitances that shunt the input and output circuit. Maximum possible gain-bandwidth product would be obtained if all external capacitances shunting the input and output circuits were reduced to zero. The expression for this maximum gain-bandwidth product provides a useful figure of merit for comparing tubes to be used in high-gain, broadband amplifiers. High figure-of-merit tubes have a high transconductance and low input and output capacitances.

6.1 Induced Currents Resulting from the Motion of Charge Between Electrodes

When an electron travels near a conductor, surface charges are induced on the conductor so that the electric field within the conductor is zero at all times. As the electron moves, the surface charges rearrange themselves to maintain zero field within the conductor. If several conductors are present in the region, and if the conductors are insulated from each other, their potentials vary with the motion of the electron. However, if two of the conductors are joined by a wire, the potential difference between them remains zero for all motions of the electron, and in general this condition can be satisfied only by a flow of charge along the wire joining the conductors.

Suppose an electron is very near one of two conductors that are joined by a wire. Practically all the lines of electric field arriving at the electron originate on positive charges on the surface of the nearby conductor. A surface charge distribution of total charge $+e$ is therefore induced on the nearby conductor. A similar situation exists when the electron is very near the other conductor. It follows, therefore, that motion of the electron from a point very near the first conductor to a point very near the second must be accompanied by a flow of charge $+e$ from the first conductor to the second through the wire joining them.

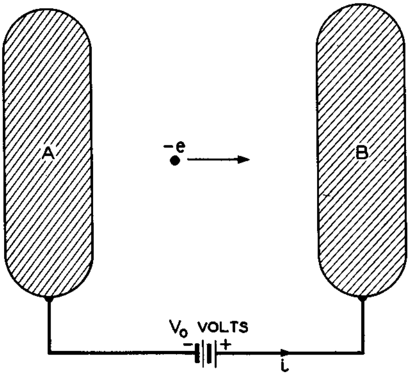


FIG. 6.1-1 An electron moving between two conductors.

Figure 6.1-1 shows an electron moving in the region between two conductors A and B. A battery maintains conductor B at a potential V_o volts above that of conductor A. If the electron travels a distance Δx in the direction of conductor B and the potential rise over the distance Δx is ΔV volts, the kinetic energy of the electron is increased by $e\Delta V$

joules, where $-e$ is the charge on the electron. This increase in kinetic energy must have come from the battery. We conclude that an amount of positive charge has flowed from conductor A to conductor B through the battery, and that the work done by the battery on the charge is $e\Delta V$. Let the amount of charge that flowed from A to B be Δq. Then

$$\Delta q V_o = e\Delta V \tag{6.1-1}$$

or

$$\Delta q = \frac{e}{V_o}\Delta V \tag{6.1-2}$$

Let Δt be the time taken by the electron in traveling the distance Δx. Dividing both sides of Equation (6.1-2) by Δt, we obtain

$$\frac{\Delta q}{\Delta t} = \frac{e}{V_o}\frac{\Delta V}{\Delta t} \tag{6.1-3}$$

This can be expressed in differential form as

$$i = \frac{dq}{dt} = \frac{e}{V_o}\left[\frac{\partial V}{\partial x}\frac{dx}{dt} + \frac{\partial V}{\partial y}\frac{dy}{dt} + \frac{\partial V}{\partial z}\frac{dz}{dt}\right] = -\frac{e}{V_o}\mathbf{E}\cdot\mathbf{u} \tag{6.1-4}$$

where i is the current that flows in the external circuit joining A and B, $-e$ is the charge on the electron, $\mathbf{E}$ is the electric field acting on the electron, and $\mathbf{u}$ is the electron velocity. We notice that $\mathbf{E}$ is proportional to V_o, so that, *for a given electron velocity* $\mathbf{u}$, *the current i is independent of the voltage applied between the electrodes.* Let us set $-\mathbf{E}(x,y,z)/V_o = \mathbf{E}_1(x,y,z)$, where $\mathbf{E}_1$ is a vector function of position having the dimensions of meters^{-1} and equal in magnitude and direction to the electric field obtained when conductor B is held 1 volt *negative* with respect to conductor A. Equation (6.1-4) can then be expressed as

$$i = e\mathbf{E}_1\cdot\mathbf{u} \tag{6.1-5}$$

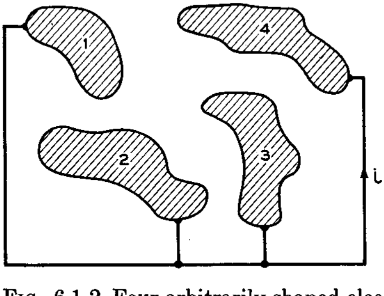


FIG. 6.1-2 Four arbitrarily shaped electrodes joined by a wire.

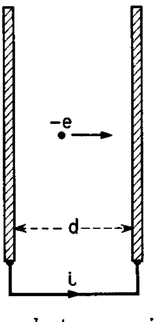


FIG. 6.1-3 An electron moving between two planar electrodes.

Since i is independent of V_o, *this must be the current that flows when the battery is omitted and the conductors are joined only by a wire or an impedance.*

Equation (6.1-5) can be applied to a system of several conductors, such as that illustrated in Figure 6.1-2. If i is the current flowing toward conductor 4, then $\mathbf{E}_1$ is a vector field having the same magnitude and direction as the electric field obtained when conductor 4 is held at a potential of -1 volt, and the remaining conductors are at ground potential.

Of particular interest is the problem of an electron moving between two planar electrodes whose linear dimensions are large compared with their spacing. Two such electrodes are illustrated in Figure 6.1-3. The electrodes are connected by a wire and are spaced by a distance of d meters. Neglecting edge effects, the vector function $\mathbf{E}_1$ is equal in magnitude to $1/d$ at all points between the electrodes, and its direction is normal to the plane of the electrodes. If the velocity of the electron is also normal to the electrodes, the current flowing in the wire is given by

$$i = \frac{eu}{d} \tag{6.1-6}$$

This result also can be obtained directly from Equation (6.1-2). For the case of the planar electrodes Equation (6.1-2) can be written as

$$\Delta q = e\frac{\Delta V}{V_o} = e\frac{\Delta x}{d} \tag{6.1-7}$$

Dividing both sides by Δt and taking the limit as $\Delta t \to 0$, Equation (6.1-6) is obtained.

If many electrons are present between the electrodes shown in Figure 6.1-1 and if they produce a charge density ρ, it follows from Equation (6.1-5) that the current in the external circuit is given by[1]

$$\begin{aligned} i &= -\int_{\text{volume}} \rho \mathbf{E}_1 \cdot \mathbf{u}\, dxdydz \\ &= -\int_{\text{volume}} \mathbf{J} \cdot \mathbf{E}_1 dxdydz \end{aligned} \tag{6.1-8}$$

where $\mathbf{J} = \rho\mathbf{u}$ is the current density at the volume element $dxdydz$, and the integral is taken over the region occupied by the space charge. This result also applies if the electrodes are joined by a wire or an impedance instead of the battery.

Next let us consider the currents that flow in the wires joining the elec-

[1]If ρ is a positive charge and moving toward electrode B, the incremental induced current $\rho\mathbf{E}_1 \cdot \mathbf{u}\, dxdydz$ flows *away* from electrode B.

trodes shown in Figure 6.1-4(a) when a single electron travels from electrode A to electrode B. We shall assume that the electron starts from rest at electrode A at time t_o. The field between the grid and electrode A accelerates the electron toward the grid, and at time t_1 it passes through an

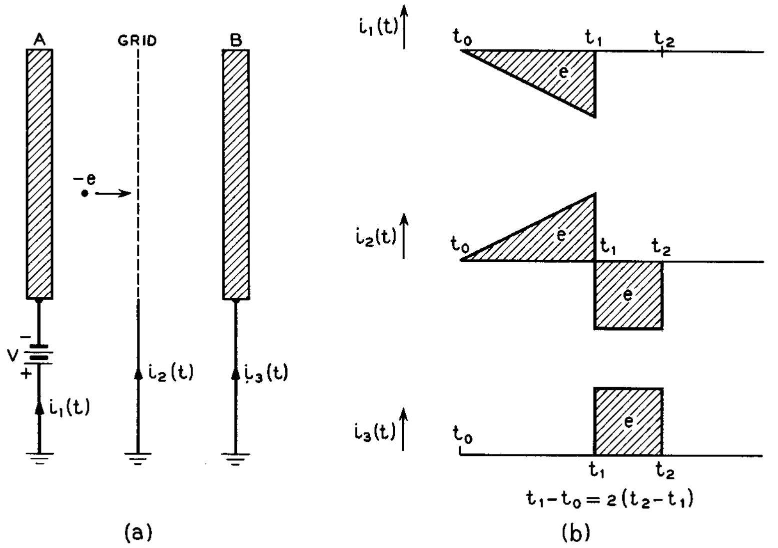


FIG. 6.1-4 Induced currents which flow in the external circuit when an electron travels from electrode A to electrode B. It is assumed that the electron starts at rest from electrode A.

opening in the grid. The electron subsequently moves with constant velocity through the second region and strikes electrode B at time t_2. In Figure 6.1-4(b) the induced currents that flow from ground toward the electrodes are plotted as functions of time. When the electron is between electrode A and the grid, it experiences a uniform accelerating field, so that its velocity and the induced current, eu/d, increase uniformly with time. To the right of the grid, where the electron travels at a steady velocity, the induced current is constant with time. The area under each of the shaded regions in Figure 6.1-4(b) is equal to the electronic charge.

While the electron is traversing the distance between electrode A and the grid, a total charge of $+e$ flows through the battery from the negative terminal to the positive terminal. This means that the battery expends eV joules of work. The work is imparted to the electron in the form of kinetic energy by the field between electrode A and the grid. When the electron strikes electrode B, its kinetic energy is dissipated in the electrode as heat. Thus the energy expended by the battery is turned into heat energy in electrode B. The positive charge that flows through the external circuit to electrode B is cancelled when the electron strikes the electrode.

6.2 Currents Induced in External Impedances

Let us consider the induced currents that flow in an external resistance connected between two electrodes. Figure 6.2-1 shows a beam of electrons which passes through a grid and strikes electrode B. Each electron is assumed to pass through the grid with the same kinetic energy. If n electrons pass through the grid per second, the same number strike electrode B per second, and a current $I_o = ne$ amperes flows from ground through the resistance to meet the arriving electrons. This current causes a voltage drop of I_oR volts in the resistance. (We assume that the voltage drop is less than the voltage through which the electrons have been accelerated, so that the electrons are not stopped before reaching electrode B.) The power dissipated in the resistance by the flow of charge is I_o^2R watts.

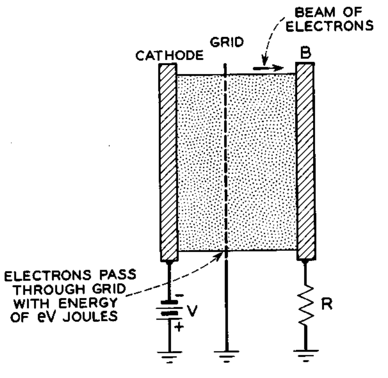


FIG. 6.2-1 Two electrodes connected by a resistance.

Because there is a voltage drop in the resistance, each electron faces a decelerating field and loses kinetic energy eI_oR joules in traveling from the grid to electrode B. Since n electrons lose this amount of kinetic energy per second, the total power lost by the electrons in traversing the region is $neI_oR = I_o^2R$ watts. But this is the power dissipated in the resistance by the positive charges flowing to meet the electrons. *Hence the kinetic energy lost by the electrons while traveling from the grid to electrode B is transformed into heat energy which is dissipated in the resistance R.* When the electrons strike electrode B, the remaining part of their kinetic energy is transformed into heat energy in electrode B.

Next let us suppose that the number of electrons passing the grid per unit time can be varied without changing their velocity. Let the current of electrons passing through the grid be given by $i = I_o + I_1\cos \omega t$. An ac voltage $I_1R \cos \omega t$ appears across the resistance, and the total instantaneous power developed in the resistance is

$$i^2R = (I_o + I_1 \cos \omega t)^2R \text{ watts} \tag{6.2-1}$$

When this is averaged over time, we obtain

$$P_{av} = I_o^2R + \frac{I_1^2R}{2} \text{ watts} \tag{6.2-2}$$

Evidently this power is greater than the dc power developed in the resistance in the absence of modulation. However, the average number of electrons passing through the grid per second is the same as in the dc case, and each has the same kinetic energy as it passes through the grid. Hence the average energy of the electrons when they strike electrode B must be less when the beam is modulated than in the dc case. To explain this, we may note that during the half cycle in which the total beam current is greater than I_o, the number of electrons passing the grid is greater than average. The voltage drop in the resistance and the retarding field are also greater than average during this half cycle. Consequently, more than half the electrons lose more kinetic energy than under dc conditions. During the other half of the cycle, less than half the electrons lose less kinetic energy than under dc conditions. Thus the ac power represented by the term $I_1^2R/2$ in Equation (6.2-2) is also obtained at the expense of the kinetic energy of the electrons.

It is easy to extend our considerations to include impedances in the external circuit. If the resistance R in the above example were replaced by an impedance Z, the instantaneous voltage across the impedance and hence the instantaneous voltage across the interelectrode space would be $\text{Re}[(I_o + I_1\epsilon^{j\omega t})Z]$, where $i = \text{Re}(I_o + I_1\epsilon^{j\omega t})$ is the instantaneous current of electrons crossing between the electrodes.

It is important to emphasize at this point that the voltage developed across the impedance is not caused by the electrons that strike electrode B flowing through the impedance. The results of Section 6.1 showed that *the induced currents flow in the external circuit only while the charge is crossing between the electrodes.* When the individual electrons strike electrode B, they cancel positive charges that have flowed to meet them.

That the induced current flowing through the impedance in the external circuit is independent of the size of the impedance is indeed a very important result. The induced current is determined only by the current of electrons crossing between the electrodes and is equal to that current. If the beam is modulated, the ac power developed in the resistive part of the load increases linearly with this resistance. Consequently, if we can modulate the beam in a manner that consumes very little power, we have a means for amplifying ac power. The ac power output, of course, is obtained at the expense of the supplies that provide the dc voltage to accelerate the electrons.

In the remainder of this chapter we shall describe the ac operation of grid-controlled tubes, and in later chapters we shall describe klystron amplifiers and traveling-wave amplifiers. We shall find that, although these tubes differ in the means used to modulate the beam and the type of load in which the ac power is developed, each provides ac power amplification by modulating an electron beam and causing ac induced currents to flow in external impedances. In klystron amplifiers and traveling-wave amplifiers the energy provided by the dc supply is first converted into kinetic energy of the electrons. This energy in turn is partly converted into ac power which is dissipated in the load and losses of the system and partly into heat energy of the electrode struck by the electrons. In grid-controlled tubes employing screen grids, a similar energy transfer occurs when the anode and screen grid are connected to the same dc supply voltage. (The screen grid and anode connections in this case would be similar to those illustrated in Figure 6.2-1.)

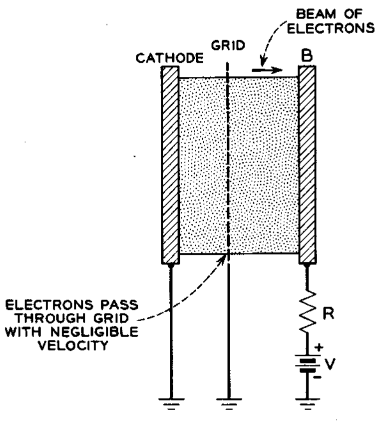


FIG. 6.2-2 Two electrodes connected by a dc supply and a resistance in series.

However, if the dc supply providing the field that accelerates the electrons is connected in series with the external impedance, as in the case of the triode tube, the transfer of power is somewhat different. In Figure 6.2-2 we show the electrodes of the previous example with a battery of V volts connected in series with the external resistance. The electrons in this case are assumed to pass through the grid with negligible velocity and are accelerated toward electrode B by the field provided by the battery. In this case the induced currents flowing through the battery and the load resistance cause some of the power expended by the battery to be transferred directly to the load resistance. The remaining power expended by the battery is converted into kinetic energy of the electrons, which in turn becomes heat energy of electrode B. If the instantaneous beam current is given by $i = I_o + I_1 \cos \omega t$, the average power developed in the resistance is $I_o^2R + I_1^2R/2$, just as in the previous example. The average power expended by the battery is I_oV, and the average power dissipated in electrode B is $I_oV - \left(I_o^2R + \frac{I_1^2R}{2}\right)$.

6.3 Equivalent Networks

Consider a grid-controlled tube in which the control-grid voltage and anode voltage are varied, while the potentials applied to the remaining electrodes are held constant. The current reaching the anode can be expressed as

$$I_{ao} = I_{ao}(V_{go}, V_{ao}) \tag{6.3-1}$$

where V_{go} and V_{ao} are the control-grid and anode voltages. Generally this function is single-valued and continuous. If I_{ao} undergoes a differential change because of differential changes in V_{go} and V_{ao}, then

$$dI_{ao} = \frac{\partial I_{ao}}{\partial V_{go}} dV_{go} + \frac{\partial I_{ao}}{\partial V_{ao}} dV_{ao} \tag{6.3-2}$$

It follows that if very small ac voltages v_g and v_a are applied to the control grid and anode, the induced ac current flowing in the anode circuit will be

$$i_a = \frac{\partial I_{ao}}{\partial V_{go}} v_g + \frac{\partial I_{ao}}{\partial V_{ao}} v_a \tag{6.3-3}$$

In terms of the tube parameters g_m and r_a discussed in Chapter 5, Equation (6.3-3) can be rewritten as

$$i_a = g_m v_g + \frac{1}{r_a} v_a \tag{6.3-4}$$

This important equation gives the induced ac current i_a which flows in the anode circuit when small ac voltages v_g and v_a are applied to the grid and anode.

Let us now consider two simple networks which we shall show to be described by Equation (6.3-4) and which can be used to simulate the tube

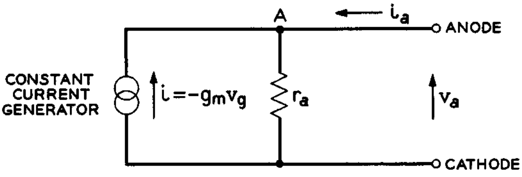


FIG. 6.3-1 The constant-current-generator small-signal equivalent network for a grid-controlled tube.

for network analysis. The first of these is illustrated in Figure 6.3-1. It involves a constant current generator,[2] which generates a current $-g_m v_g$, in parallel with a resistance equal to the dynamic anode resistance r_a. Referring to the figure, it is evident that the current flowing through the resistance r_a away from the point A is $-g_m v_g + i_a$. This current, multi-

[2]The symbol used in Figure 6.3-1 for a constant current generator will be used in subsequent illustrations in this and later chapters. Likewise, the symbol used in Figure 6.3-3 for a constant voltage generator will be used in subsequent illustrations in this and later chapters.

plied by the resistance r_a, must equal the voltage v_a applied between the terminals. Hence

$$v_a = (-g_m v_g + i_a)r_a \qquad (6.3\text{-}5)$$

A simple rearrangement of this equation shows that it is just another form of Equation (6.3-4). Thus, when a voltage v_a is applied between the terminals of the network shown in Figure 6.3-1, a current i_a, given by Equation (6.3-4), flows through the network from one terminal to the other. The same current flows in the anode lead of a grid-controlled tube when ac voltages v_g and v_a are applied to the control grid and anode. Consequently, the network shown in Figure 6.3-1 can be used in place of the tube for purposes of network analysis. Figure 6.3-2(b) shows the construction of an

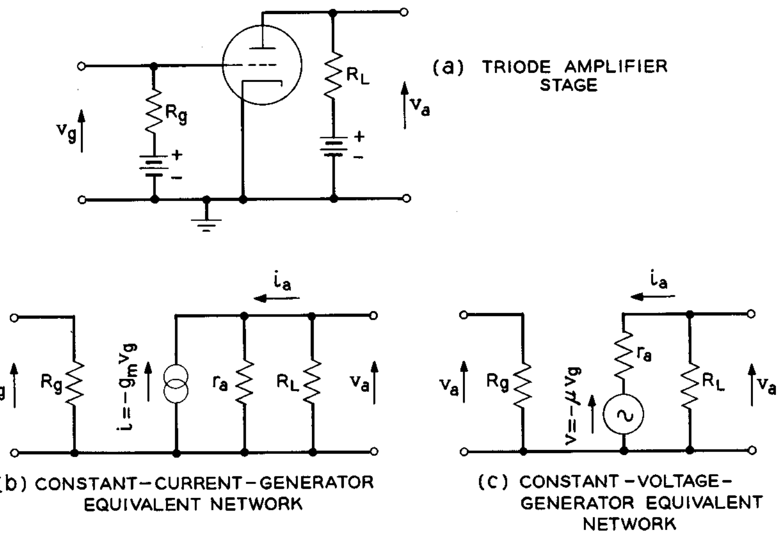


FIG. 6.3-2 A grounded-cathode triode amplifier stage and two low-frequency, small-signal equivalent networks.

equivalent network for analysis of the low-frequency response of the simple grounded-cathode triode amplifier stage shown in Figure 6.3-2(a). Constant current generators always have infinite internal impedance. This means that the voltages applied across the terminals of the generator do not affect the current generated.

In constructing the equivalent network shown in Figure 6.3-2(b), we have assumed that the control grid of the triode has a sufficiently negative bias that it does not draw any current. Secondly, since Equation (6.3-4) is valid only when the amplitudes of the ac signals are small, the equivalent network is suitable only for analysis of the small-signal operation of the stage. Finally, since we have not accounted for interelectrode capacitances, interwiring capacitances, and lead inductances, the equivalent network can be used only at sufficiently low frequencies that these capacitances and inductances can be neglected.

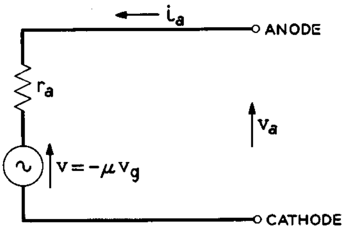


FIG. 6.3-3 The constant-voltage-generator small-signal equivalent network for a grid-controlled tube.

The second network described by Equation (6.3-4) is shown in Figure 6.3-3. It involves a constant voltage generator, which generates a voltage $-\mu v_g$, in series with a resistance equal to the dynamic anode resistance r_a. By equating the sum of the voltages around the loop in this network to zero, we obtain.

$$v_a - i_a r_a + \mu v_g = 0 \tag{6.3-6}$$

Substituting $\mu = g_m r_a$ into this, we see that this equation is also equivalent to Equation (6.3-4). Thus the network shown in Figure 6.3-3 also can be used to simulate the ac response of a grid-controlled tube with ac voltages v_g and v_a applied to the control grid and anode. Figure 6.3-2(c) shows the construction of a constant-voltage-generator equivalent network for the triode stage in Figure 6.3-2(a). Constant voltage generators always have zero internal impedance.

If the triode in Figure 6.3-2(a) were replaced by a tetrode or pentode and the additional electrodes were maintained at constant potentials, the equivalent networks shown in Figures 6.3-2(b) and 6.3-2(c) would still be applicable at low frequencies. However, since μ and r_a tend to be extremely high for tetrodes and pentodes, the constant-current-generator equivalent network is usually found more satisfactory for the analysis of stages using these tubes.

The voltage gain of the amplifier stage shown in Figure 6.3-2 is given by the magnitude of the ratio of the ac voltage developed across the load resistance R_L to the input voltage v_g. From the figure it can be seen that this ratio is given by

$$\text{voltage gain} = \left|\frac{v_a}{v_g}\right| = \frac{\mu R_L}{r_a + R_L} = g_m \frac{r_a R_L}{r_a + R_L} \tag{6.3-7}$$

In stages employing tetrodes and pentodes, r_a may be very large compared with R_L, in which case the gain is very nearly given by

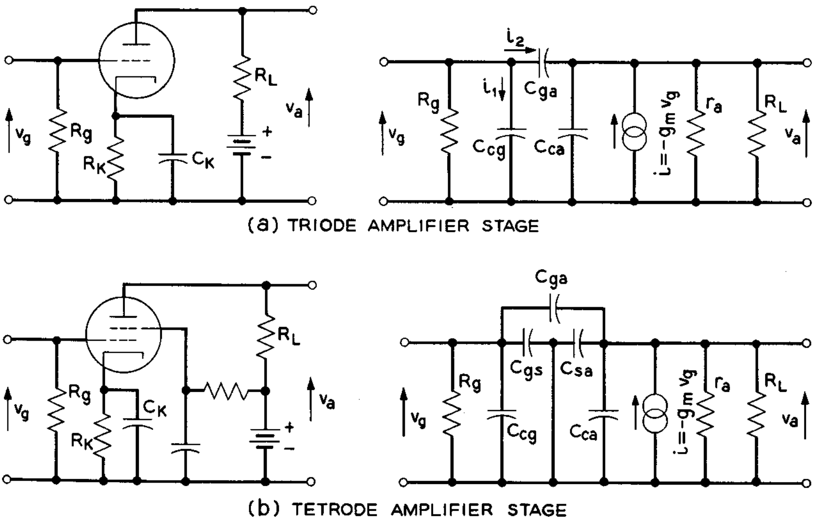
$$\text{voltage gain} = g_m R_L \tag{6.3-8}$$

Next let us extend the equivalent network shown in Figure 6.3-2(b) to include the effects of interelectrode capacitances and stray wiring capacitances. In the 417A triode described in Sections 5.1 and 5.2, the interelectrode capacitances are:

	pf*
Grid to-anode capacitance	1.5
Grid-to-cathode capacitance	5.4
Anode-to-cathode capacitance	0.2

*One picofarad = 10^{-12} farad.

These capacitances include those between the internal leads to the electrodes. In addition, the stray capacitances associated with external wiring and other circuit components may easily amount to 4 or 5 pf both between the grid circuit and ground and between the anode circuit and



C_{cg} = CATHODE-CONTROL GRID CAPACITANCE PLUS STRAYS
C_{ca} = CATHODE-ANODE CAPACITANCE PLUS STRAYS
C_{sa} = SCREEN-ANODE CAPACITANCE
C_{ga} = CONTROL GRID-ANODE CAPACITANCE
C_{gs} = CONTROL GRID-SCREEN CAPACITANCE

FIG. 6.3-4 Small-signal equivalent networks for a triode amplifier stage and a tetrode amplifier stage. The networks include the effects of interelectrode capacitance and stray capacitance.

ground. At frequencies above 100 kc the reactances associated with these capacitances may be comparable with other circuit impedances and consequently they must be considered in the circuit analysis. Figure 6.3-4 shows equivalent networks for simple triode and tetrode amplifier stages where the interelectrode and stray capacitances are taken into account. In both networks we have assumed that the reactance of the capacitance C_K is small compared with R_K and therefore that both quantities can be neglected.

Let us use the equivalent network shown in Figure 6.3-4(a) to determine the input admittance of the triode stage shown in the figure. This admittance is given by

$$Y_i = \frac{1}{R_g} + \frac{I_1 + I_2}{V_g} \tag{6.3-9}$$

where I_1 is the phasor corresponding to the current i_1 which flows through the cathode-to-grid capacitance C_{cg} (that is, $i_1 = \text{Re}\, I_1\epsilon^{j\omega t}$, where ω is the angular frequency of the signal), I_2 is the phasor corresponding to the current i_2 which flows through the grid-to-anode capacitance C_{ga}, and V_g is the phasor corresponding to the input voltage v_g. Now $I_1 = V_g\, j\omega C_{cg}$, and $I_2 = (V_g - V_a)\, j\omega C_{ga}$, where V_a is the phasor corresponding to the output voltage v_a. Substituting these expressions into Equation (6.3-9), we obtain

$$Y_i = \frac{1}{R_g} + j\omega\left[C_{cg} + \left(1 - \frac{V_a}{V_g}\right)C_{ga}\right] \tag{6.3-10}$$

The ratio $-V_a/V_g$ is the complex gain of the stage. Since this is likely to have a large, positive real part, the input signal may be shunted by a large apparent capacitance. Because C_{ga} is much smaller in tetrodes and pentodes, the shunting capacitance given by Equation (6.3-10) is greatly reduced in stages employing these tubes. In Section 6.4 we shall see that a low input capacitance is needed for tubes used in high-gain, broadband amplifiers. For this reason, most multistage high-gain, broadband amplifiers employ tetrode or pentode tubes. However, triodes are sometimes used in the input stages of these amplifiers because of their better noise performance. (See Chapter 13.)

Finally, let us determine the low-frequency input admittance of the grounded-grid amplifier stage shown in Figure 6.3-5(a). A low-frequency equivalent network for the stage is shown in part (b) of the figure. When a small ac voltage v_c is applied to the cathode, an ac current i_a flows in the cathode and anode leads of the tube, and an ac current i_1 flows in the resistance r_a of the equivalent network. We neglect the effects of the anode-

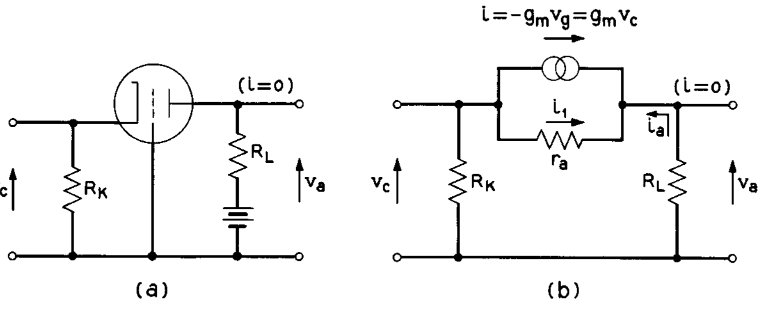


FIG. 6.3-5 A grounded-grid amplifier stage and its low-frequency equivalent network.

to-cathode capacitance in this analysis. Applying Kirchhoff's Laws to the network, we obtain

$$I_a = -(g_m V_c + I_1) \tag{6.3-11}$$

and

$$V_c = I_1 r_a - I_a R_L \tag{6.3-12}$$

where I_a, I_1, and V_c are the phasors corresponding to i_a, i_1, and v_c. These equations can be solved for I_a/V_c, and the input admittance can be expressed as

$$Y_i = \frac{1}{R_K} - \frac{I_a}{V_c} = \frac{1}{R_K} + \frac{g_m + 1/r_a}{1 + R_L/r_a} \tag{6.3-13}$$

If $r_a \gg R_L$ and $\mu \gg 1$, this reduces to

$$Y_i = \frac{1}{R_K} + g_m \tag{6.3-14}$$

If the tube has a transconductance of 0.025 mho, the input impedance is 40 ohms in parallel with R_K. Grounded-grid amplifier stages are of limited usefulness because of their low input impedance.

6.4 Gain-Bandwidth Product

Resonant circuits are used in high-frequency amplifier stages to establish the frequency at which maximum gain is obtained. Because the gain falls off on either side of this frequency, each stage can be characterized by a bandwidth, or a range of frequencies about the resonant frequency over which the gain of the stage is within certain limits. Usually these limits are expressed as a number of db below the maximum gain. Thus the "3-db bandwidth" of an amplifier stage is the range of frequencies over which the power gain is within 3 db of the maximum gain. Since 3 db very nearly corresponds to a power ratio of 2 and a voltage ratio of $\sqrt{2}$, the voltage gain at the extreme frequencies of the 3-db bandwidth is $1/\sqrt{2}$ times that at maximum gain.

In this section we show that the product of gain and bandwidth for an amplifier stage employing resonant circuits is a constant which depends only upon parameters of the tube itself and upon the external capacitances shunting the tube. By changing the load resistance in the anode circuit, both the gain of the stage and the bandwidth change in such a manner that the product of gain and bandwidth remain unchanged. First, it will be helpful to examine a few properties of resonant circuits.

If a parallel-resonant circuit is excited by an external source and then allowed to oscillate freely, the excitation energy is stored alternately in the electric field of the capacitance and in the magnetic field of the inductance. As the oscillation continues, the losses of the circuit cause the amplitude of the oscillation to decrease. Although there is always some capacitance and resistance between the terminals of the inductance, and some inductance and resistance between the terminals of the capacitance, for most purposes the circuit can be assumed to consist of a pure inductance, a pure capacitance, and a pure resistance, all in parallel. Such a circuit is shown in Figure 6.4-1. The resistance R is assumed to be of a magnitude which accounts for losses in the inductance and capacitance, as well as any additional resistance which is connected in parallel with the circuit. The magnitude of the admittance of the parallel combination is given by

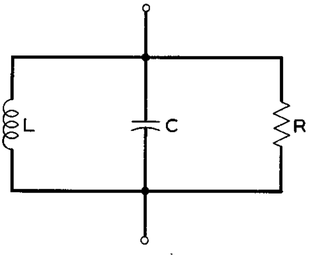


FIG. 6.4-1 A parallel resonant circuit.

$$|Y| = \sqrt{(1/R)^2 + (\omega C - 1/\omega L)^2} \tag{6.4-1}$$

where ω is the angular frequency of the exciting signal. If the circuit is excited by a constant current source which generates a current $I_o \sin \omega t$, a sinusoidal voltage with amplitude $I_o/|Y|$ is developed across the circuit. Resonance occurs when $\omega C = 1/\omega L$, or $\omega = 1/\sqrt{LC}$. At this frequency $|Y|$ is a minimum and equal to $1/R$.

A measure of the quality of a parallel resonant circuit is given by the Q of the circuit, which is defined as

$$Q = 2\pi \frac{\text{energy stored at resonance}}{\text{energy lost per cycle}} \tag{6.4-2}$$

Since the energy stored in a capacitance C which is charged to a voltage V is $\frac{1}{2} CV^2$, it is easily shown that, for a parallel resonant circuit, Q is given by

$$Q = 2\pi \frac{\frac{1}{2} C V_{\max}^2}{\frac{V_{\max}^2}{2R} \cdot \frac{1}{f_o}} = \omega_o CR = \frac{R}{\omega_o L} \tag{6.4-3}$$

where $V_{\max}$ is the peak voltage appearing across the circuit, and $\omega_o = 2\pi f_o$ is the angular frequency of resonance.

From Equation (6.4-1) it follows that, if the circuit is excited by an ac current of constant amplitude but variable frequency, the voltage developed across the circuit falls to $1/\sqrt{2}$ of its maximum value when the angular frequency ω is such that

$$\left| \omega C - \frac{1}{\omega L} \right| = \frac{1}{R} \tag{6.4-4}$$

Multiplying both sides of this equation by R/Q and substituting for R/Q from Equation (6.4-3), we obtain

$$\left| \frac{\omega}{\omega_o} - \frac{\omega_o}{\omega} \right| = \frac{1}{Q} \tag{6.4-5}$$

Rearranging this gives

$$\left| \frac{(\omega - \omega_o)(\omega + \omega_o)}{\omega \omega_o} \right| = \frac{1}{Q} \approx \frac{2\Delta f}{f_o} \tag{6.4-6}$$

or

$$Q \approx \frac{f_o}{2\Delta f} \tag{6.4-7}$$

where Δf is the number of cycles away from resonance at which the voltage across the circuit falls to $1/\sqrt{2}$ of its maximum value. The frequency $2\Delta f$ gives a measure of the width of the resonance response of the tuned circuit when excited by a constant current generator.

Let us now use these parallel-resonant-circuit concepts to determine the gain-bandwidth product for a pentode amplifier stage. The amplifier stage is shown in Figure 6.4-2, together with its equivalent network. The capacitance C_i in the equivalent network is the input capacitance of the tube, or the sum of the capacitances between the control grid and all other electrodes except the anode. C_o is the output capacitance of the tube, or the sum of the capacitances between the anode and all other electrodes except the control grid. C_s is the sum of the stray capacitances shunting the output circuit plus any lumped capacitances that are connected across the output circuit. We shall assume that the capacitance C_{ga} between the

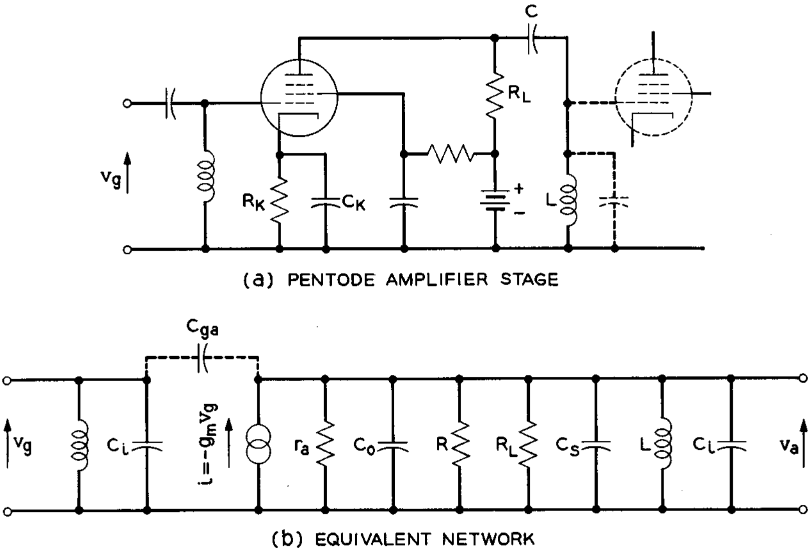


FIG. 6.4-2 A pentode amplifier stage with a parallel resonant circuit connected to the anode.

control grid and anode is sufficiently small that it can be neglected. Also, the coupling capacitance C is assumed to be large enough that it has negligible reactance at the frequencies under consideration. The resistance R accounts for the losses in the output circuit, including those in the inductance and capacitances. Usually it will be much larger than the load resistance R_L.

Maximum gain for the stage occurs at the frequency at which the inductance L resonates with the capacitances in parallel with it. If we let R_e be the equivalent resistance of r_a, R, and R_L in parallel, the maximum gain of the amplifier stage is given by

$$\text{gain} = g_m R_e \tag{6.4-8}$$

Since r_a and R are likely to be large compared with R_L, this is very nearly equal to $g_m R_L$. The Q of the tuned circuit shunted by the resistance R_e is given by

$$Q = \omega_o(C_o + C_i + C_s)R_e \tag{6.4-9}$$

where C_i is the input capacitance of the following stage. From Equation (6.4-7), the bandwidth over which this stage gives at least $1/\sqrt{2}$ of the maximum voltage gain is given by

$$2\Delta f = \frac{f_o}{Q} = \frac{1}{2\pi(C_o + C_i + C_s)R_e} \tag{6.4-10}$$

Finally, the product of gain and bandwidth for the stage is obtained by multiplying Equations (6.4-8) and (6.4-10). Thus,

$$\text{gain} \times \text{bandwidth} = \frac{g_m}{2\pi(C_o + C_i + C_s)} \qquad (6.4\text{-}11)$$

If the stray capacitances were zero and there were no lumped capacitances shunting the output circuit, the expression would reduce to

$$\text{gain} \times \text{bandwidth} = \frac{g_m}{2\pi(C_o + C_i)} \qquad (6.4\text{-}12)$$

The product given by Equation (6.4-12) represents a theoretical limit that is not attained in practice because of the stray capacitances that are always present. If we assume that the input capacitance C_i of the tube in the next stage is the same as that of the tube under consideration, the gain-

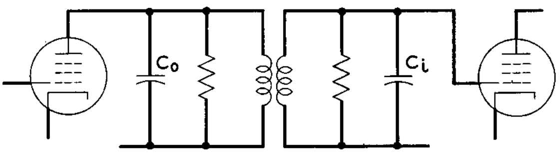


FIG. 6.4-3 A double-tuned resonant circuit between amplifier stages.

bandwidth product given by Equation (6.4-12) depends entirely on the parameters of the tube itself, and not upon R_L or L. The equation shows that we can design the stage to have high gain at the expense of narrow bandwidth, or lower gain and a larger bandwidth. High gain, of course, is associated with large R_L. It should be noted that R_L includes the effect of the input loading of the following stage (see Section 7.3), and in practice this loading places an upper limit on the maximum gain per stage.

Sometimes double-tuned resonant circuits are employed between amplifier stages, as illustrated in Figure 6.4-3. For such a circuit the gain-bandwidth product depends upon the degree of coupling between the two circuits and upon the Q's of the circuits. If the circuits are adjusted for "critical coupling" and if the Q's of the primary and secondary circuits are equal, it can be shown[3] that the gain-bandwidth product of the stage is given by

$$\text{gain} \times \text{bandwidth} = \frac{g_m}{2\pi\sqrt{2}\,\sqrt{C_i C_o}} \qquad (6.4\text{-}13)$$

where C_i is the total input capacitance of the following stage, including strays, and C_o is the total output capacitance, including strays.

[3]Reference 6b.

Equations (6.4-12) and (6.4-13) tell us that tubes to be used in high-gain, broadband amplifiers should have high transconductance and low input and output capacitances. Unfortunately, increasing the transconductance by increasing the cathode area also increases both the input and output capacitances, and if the transconductance is increased by decreasing the cathode-to-grid spacing, this also increases the input capacitance. However, in practice the stray capacitances which shunt the input circuit represent an appreciable part of the input capacitance, so that, if the tube design is changed to double the cathode area and hence double the transconductance, it is likely that the total input capacitance including strays will not be doubled, the stray capacitance being nearly constant. Furthermore, when double-tuned circuits are employed, the gain-bandwidth product increases directly with the transconductance but depends only on the reciprocal of the square root of the input capacitance. For these reasons, tubes for use in high-gain, broadband amplifiers are designed to have high transconductance. A large spacing between the anode and screen grid or suppressor grid is also desirable, since the output capacitance C_o is then reduced.

Equation (6.4-12) is frequently used to give a figure of merit for comparing tubes for use in high-gain, broadband amplifier applications.[4] By using a tube with a high gain-bandwidth product in a multistage amplifier, less stages are needed to achieve a total over-all gain and bandwidth. Table 5.2-1 lists the maximum-possible-gain-bandwidth product $g_m/2\pi(C_o + C_i)$ for the 448A tetrode and 403A/6AK5 pentode as 215 and 95 Mc, respectively. In practice, stray capacitance amounting to a total of 9 pf might shunt the input and output circuits, and in this case the *actual* gain-bandwidth products would be reduced to 160 Mc and 46 Mc for stages using these tubes.

PROBLEMS

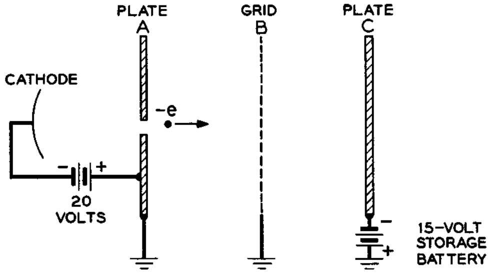
6.1 Show that for a space-charge-limited planar diode the current pulse induced in the external circuit by the passage of a single electron from the cathode to the anode is given by

$$i = \frac{3et^2}{\tau^3}$$

where it is assumed that the potential minimum coincides with the cathode, τ is the length of the current pulse, and t is the time measured from the instant the electron passes the potential minimum. Hint: Show that i is proportional to t^2 without

[4]Sometimes Equation (6.4–13) is used.

calculating the exact value of the constant of proportionality, then use the fact that $\int_0^\tau i\,dt = e$ to find the value of this constant.

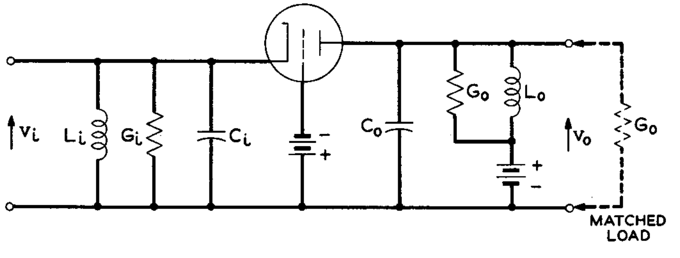


Problem 6.2

6.2 In the apparatus shown, a single electron leaves the cathode and is accelerated toward plate *A*. It passes through the hole in plate *A* with 20 electron volts of kinetic energy and travels on through grid B to strike plate *C*.

(a) Sketch the current that flows from ground toward grid *B* as the electron travels from plate *A* to plate *C*. Indicate the relative values of the induced current at the times when the electron is at electrodes *A*, *B*, and *C*.

(b) When the electron has struck plate *C*, where can the 20 electron volts of kinetic energy that the electron had when it passed through plate *A* be found?



Problem 6.3

6.3 The figure shows a grounded-grid triode amplifier stage. C_i and C_o are, respectively, the input and output capacitances of the tube plus strays. (Notice that in this case the input capacitance is the cathode-to-grid capacitance, and the output capacitance is the grid-to-anode capacitance.) The input and output inductance L_i and L_o resonate with C_i and C_o, respectively, at the same frequency. The conductance G_i and G_o take into account the effects of losses in the input and output resonant circuits.

(a) Sketch the constant current generator equivalent network for the stage. (Neglect the cathode-to-anode capacitance.)

(b) The power gain of the stage can be defined as

$$\frac{\text{power delivered to a matched load with conductance } G = G_o}{\text{power dissipated in input circuit}}$$

Show that the power gain at resonance is given by $g_m/4G_o$. Assume that the dynamic anode resistance r_a is sufficiently large that very little of the current from the constant current generator flows through r_a.

(c) Show that the product of the power gain at resonance and the 3-db bandwidth for the stage is $g_m/4\pi C_o$, where the 3-db bandwidth is the bandwidth between frequencies at which the power gain has dropped 3 db below maximum gain.

6.4 In an amplifier stage such as that shown in Figure 6.3-4(a) the gain falls off with increasing frequency because of the capacitance shunting the output circuit. Show that the product of the zero-frequency voltage gain and bandwidth over which the gain is within 3 db of the zero-frequency gain is given by $g_m/2\pi C$, where C is the total capacitance shunting the output. Neglect the effect of the control grid-to-anode capacitance.

REFERENCES

The induced currents that flow in the circuit joining two electrodes when charge moves in the space between the electrodes is discussed in the following reference:

6a. W. Shockley, *J. Appl. Phys.* **9**, 635, 1938.

A thorough treatment of amplifiers employing grid-controlled tubes is given in the following reference:

6b. *Vacuum Tube Amplifiers, MIT Radiation Laboratory Series*, Vol. 18, McGraw-Hill Book Co., Inc., New York, 1948.